FULL-WAVE MODAL ANALYSIS OF ARBITRARILY-SHAPED DIELECTRIC WAVEGUIDES THROUGH AN EFFICIENT BOUNDARY-ELEMENT-METHOD FORMULATION

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Abstract

In this work an original procedure, based on the boundary element method (BEM), is carried out for the full-wave modal analysis of dielectric waveguiding structures with arbitrary cross section. A new advantageous integral-equation formulation is reached after a careful analysis of the dyadic kernel's discontinuities. Numerical solutions are then derived by means of both conventional and novel algorithms. Various results for important microwave applications, compared to data from other numerical approaches and from measurements, emphasize the notable accuracy and efficiency of such implementation.

Introduction

The electromagnetic characterization of dielectric waveguiding structures is an argument of remarkable theoretical and practical interest, both for microwave and for optics applications [1,2]. The modal properties (propagation wavenumbers, field configurations, etc.) of many different types of dielectric guides have widely been analyzed in the literature. In particular, for analytically non-solvable structures, a good deal of numerical techniques have been proposed and discussed as regards the characteristics of computing accuracy, versatility, efficiency, etc. [3].

In this work, based on the boundary element method (BEM), a new procedure is developed and applied to solve in a complete, accurate, and efficient way the modal problem for cylindrical dielectric structures having arbitrary cross section.

As is known, on the one hand, the reduction of complexity of one spatial dimension, which is typical of BEM approaches, appears extremely convenient in terms of memory space and computing time [4]; on the other hand, problems could derive from rather involved pre-processing and numerical convergency.

Compared with the BEM formulations already outlined in the literature [5-7], the procedure that has here been developed will present some important advantageous distinctive features. In particular, through the present formulation it is possible to reach a great flexibility in the choice of the basis functions for the unknowns, thus enlarging significantly the class of algorithms for the numerical solution of the integral equations. To this aim, to eliminate some involved numerical problems, a deep analysis on the singularities of the integral-equation kernels (represented by dyadic Green's functions) becomes necessary. The numerical solution is then obtained through different discretization techniques, both usual (point matching and Galerkin's methods [4,6]) and unusual (Nystrom's method [8]), which are tested as concerns their accuracy and efficiency properties.

This BEM procedure provides a new tool for full-wave modal analysis of a variety of dielectric structures of practical interest. Examples are here given concerning different shapes of nonradiative dielectric (NRD) resonators [9]. The accuracy and economy of the BEM results are compared to data derived by other numerical techniques and measurements. Specific attention is also paid to problems that are extremely delicate from a computational viewpoint: for instance, the effects of slight perturbations in the cross section shape (e.g., notches or cuts) are tested, thus deriving useful information for design of devices such as filters of dual-mode type [10].

Description of the procedure of analysis

The boundary element method is here applied with a view of obtaining the complete spectrum of guided modes for dielectric structures of cylindrical type with arbitrary cross section, as schematized in Fig. 1.

The basic formulation of the problem is based on the equivalence principle, by expressing the fields in the interior and exterior of the cylinder by means of free-space dyadic Green's functions, related to the different media which occupy the two regions. By imposing the continuity of the unknown tangential components of the electric and magnetic fields on the air/dielectric interface, a couple of integral equations on the separation surface is obtained.

According to the standard approaches, the longitudinal symmetry shown by the structures of interest suggests a common z-dependence $exp(-j\beta z)$ for the unknowns. The integration along z furnishes a Fourier transform and the problem becomes two-dimensional. On the generic cross section the position vector will be represented by $\mathbf{r}_t$ (the apex will be referred to source points); on the boundary s of the section (where the integrations are extended), a local rectangular coordinate system $\mathbf{n}_0,\mathbf{l}_0,\mathbf{z}_0$ (normal, tangential, and longitudinal unit vectors, respectively) may be chosen, as shown in Fig. 1.

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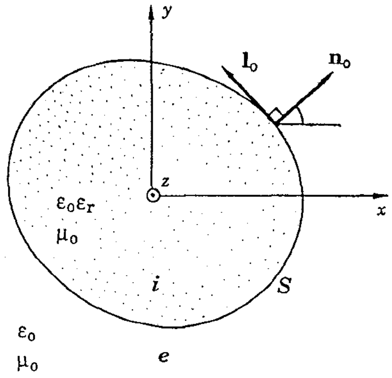


Fig. 1. Dielectric waveguiding structure of cylindrical type with arbitrary cross section, analyzed with the boundary element method (BEM). The local coordinate system on the boundary s and the other introduced parameters are indicated.

The fields in the external (apex e) and internal (apex i) regions can be expressed in the following integral forms (we present for brevity only the expression for the electric field, since it is possible to immediately deduce the magnetic field by means of the duality principle):

$$\mathbf{E}^{e/i}(\mathbf{r}_t) = -/+\, j\omega\mu_o \oint_s \mathbf{J}_e(\mathbf{r}'_t) \cdot \overline{\mathbf{G}}_t^{e/i}(\mathbf{r}'_t, \mathbf{r}_t; \beta)\, ds'$$

$$-/+ \oint_s \mathbf{J}_m(\mathbf{r}'_t) \cdot \left(\nabla'_t + j\beta\mathbf{z}_o\right) \times \overline{\mathbf{G}}_t^{e/i}(\mathbf{r}'_t, \mathbf{r}_t; \beta)\, ds'$$

where the equivalent electric and magnetic currents $\mathbf{J}_e$ and $\mathbf{J}_m$ are defined as:

$$\mathbf{J}_e = \mathbf{n}_o \times \mathbf{H} = \mathbf{z}_o H_l - \mathbf{l}_o H_z = \mathbf{z}_o J_{ez} - \mathbf{l}_o H_z$$
$$\mathbf{J}_m = -\mathbf{n}_o \times \mathbf{E} = -\mathbf{z}_o E_l + \mathbf{l}_o E_z = \mathbf{z}_o J_{mz} + \mathbf{l}_o E_z$$

and the transformed free-space dyadic Green's functions $\overline{\mathbf{G}}_t^{e/i}$ are deduced from the two-dimensional scalar Green's function g_t:

$$\overline{\mathbf{G}}_t^{e/i}(\mathbf{r}'_t, \mathbf{r}_t; \beta) = \left[\overline{\mathbf{I}} + \frac{1}{k^2}\left(\nabla'_t + j\beta\mathbf{z}_o\right)\left(\nabla'_t + j\beta\mathbf{z}_o\right)\right] g_t^{e/i}(\mathbf{r}'_t, \mathbf{r}_t)$$

$$g_t^e(\mathbf{r}'_t, \mathbf{r}_t) = \frac{1}{4j} H_o^{(2)}\left(k_{to}\left|\mathbf{r}'_t - \mathbf{r}_t\right|\right)$$
$$g_t^i(\mathbf{r}'_t, \mathbf{r}_t) = \frac{1}{4j} H_o^{(2)}\left(k_{t\varepsilon}\left|\mathbf{r}'_t - \mathbf{r}_t\right|\right)$$

where: $k_{to}^2 = k_o^2 - \beta^2$, $k_{t\varepsilon}^2 = k_o^2\varepsilon_r - \beta^2$, and k is the wave-number of the medium in which the corresponding Green's function is calculated: i.e., $k = k_o$ in the air (e region) and $k = k_o\sqrt{\varepsilon_r}$ in the dielectric (i region).

In spite of the relative simplicity of the BEM formulation, the effective solution of the integral-equation system presents considerable difficulties, both from the analytical and the numerical point of view.

In connection with these questions, it should be pointed out that in the BEM procedures already presented in the literature [5-7], the operations involving derivatives on the Green's functions are usually transferred to the unknowns, so that the singularity degree of the integral kernels is reduced. In the present formulation, on the contrary, we have preferred to maintain the derivative operations on the Green's functions in order to make the choice of the basis functions for the unknowns' expansion more flexible. As a consequence, this choice allows us to significantly enlarge the class of the numerical techniques that can be used for faster and more accurate solutions (this argument is treated in detail in the next section).

The advantages of such a formulation require, on the other hand, a large amount of analytical processing: in particular, a specific attention has to be paid in evaluating the influence of the kernel's singularities, whose degree is no more reducible. Since the dyadic Green's functions diverge when the source and the observation points coincide on the boundary s, the integrals have to be evaluated in the limit for which the observation point tends to the boundary from the appropriate (external or internal) region. If the boundary s is supposed to be represented through a polygonal contour, this procedure of limit is applicable in a rigorous way: it leads to suitable expressions for the external and internal electromagnetic fields, which are represented by the sum of various addenda. One of such addenda requires the operation of 'finite part' on an integral [6]: this contribution is the most delicate to be calculated in a numerical way, and therefore we have suitably combined the equations to eliminate such a highly singular term.

Based on these considerations, we have reached the following reference expression for the integral-equation system, from which the numerical solution has been derived:

$$\frac{\begin{Bmatrix}\varepsilon_r\\1\end{Bmatrix}+1}{2}\,\mathbf{n}_o \times \begin{Bmatrix}\mathbf{E}(\mathbf{r}_t)\\\mathbf{H}(\mathbf{r}_t)\end{Bmatrix} + \mathbf{n}_o \times \Big\{ \oint_s \begin{Bmatrix}J_{mz}(\mathbf{r}'_t)\\-J_{ez}(\mathbf{r}'_t)\end{Bmatrix}$$

$$\cdot\Big[\frac{\partial}{\partial n'}\Big(g_t^e - \begin{Bmatrix}\varepsilon_r\\1\end{Bmatrix} g_t^i\Big)\mathbf{l}'_o - \frac{\partial}{\partial l'}\Big(g_t^e - \begin{Bmatrix}\varepsilon_r\\1\end{Bmatrix} g_t^i\Big)\mathbf{n}'_o\Big]ds' + \oint_s \begin{Bmatrix}E_z(\mathbf{r}'_t)\\H_z(\mathbf{r}'_t)\end{Bmatrix}$$

$$\cdot\Big[-\frac{\partial}{\partial n'}\Big(g_t^e - \begin{Bmatrix}\varepsilon_r\\1\end{Bmatrix} g_t^i\Big)\mathbf{z}_o + j\beta\Big(g_t^e - \begin{Bmatrix}\varepsilon_r\\1\end{Bmatrix} g_t^i\Big)\mathbf{n}'_o\Big]ds' + j\omega\begin{Bmatrix}\mu_o\\\varepsilon_o\end{Bmatrix}$$

$$\cdot\oint_s \begin{Bmatrix}J_{ez}(\mathbf{r}'_t)\\J_{mz}(\mathbf{r}'_t)\end{Bmatrix}\Big[j\frac{\beta}{k_o^2}\frac{\partial}{\partial n'}(g_t^e - g_t^i)\,\mathbf{n}'_o + j\frac{\beta}{k_o^2}\frac{\partial}{\partial l'}(g_t^e - g_t^i)\,\mathbf{l}'_o$$

$$+ (g_t^e - \varepsilon_r g_t^i)\,\mathbf{z}_o - \frac{\beta^2}{k_o^2}(g_t^e - g_t^i)\,\mathbf{z}_o\Big]ds' - j\omega\begin{Bmatrix}\mu_o\\\varepsilon_o\end{Bmatrix}\oint_s \begin{Bmatrix}H_z(\mathbf{r}'_t)\\-E_z(\mathbf{r}'_t)\end{Bmatrix}$$

$$\cdot\Big[\frac{1}{k_o^2}\frac{\partial^2}{\partial n'\partial l'}(g_t^e - g_t^i)\mathbf{n}'_o + (g_t^e - \varepsilon_r g_t^i)\mathbf{l}'_o + j\frac{\beta}{k_o^2}\frac{\partial}{\partial l'}(g_t^e - g_t^i)\,\mathbf{z}_o\Big]ds'$$

$$-j\omega\begin{Bmatrix}\mu_o\\\varepsilon_o\end{Bmatrix}\oint_s \begin{Bmatrix}H_z(\mathbf{r}'_t)\\-E_z(\mathbf{r}'_t)\end{Bmatrix}\frac{1}{k_o^2}\frac{\partial^2}{\partial l'^2}(g_t^e - g_t^i)\,\mathbf{l}'_o\, ds'\Big\} = 0$$

All the terms of this pair of equations can be integrated and evaluated numerically everywhere, except for a neighborhood of the source point for which the integration has been led analytically without approximations.

Procedures for the numerical solution

The integral-equation expression that has here been obtained describes rigorously the electromagnetic eigenvalue problem for any dielectric waveguiding structure whose contour can be reduced to a polygonal. As previously discussed, the present formulation permits us to enlarge and improve the numerical techniques for the solution.

By following the most common approach known in the literature, at first the equation system has here been solved through the method of moments (MoM) [4,6]. Since here the derivative operations on the unknowns have on purpose been avoided, the most effective choice for their expansion appears to be the linear combination of basis functions that are piece-wise constant.

The easiest way for operating the testing is given by a point matching, for which the continuity of the tangential components is enforced on a number of points (chosen in the middle of each integration sub-interval) equal to the number of basis functions used for the unknowns' expressions. A solution approach has been developed also with Galerkin's method, where the testing functions are equal to the basis functions, thus requiring a double integration through standard techniques (Gauss methods) [8].

An interesting alternative procedure, which is not applicable to the formulations already presented in the literature (since the absence of derivations on the unknowns is requisite), has consisted in an adaptation of the integral-equation solution method due to Nystrom, which is based on quadrature formulas [8]. Such technique neither requires any set of basis functions for the unknowns, nor developments of numerical integrations on the boundary. Proper tests and comparisons among the different numerical techniques that have here been employed have emphasized that Nystrom's method, by virtue of the described advantages, generally presents characteristics of calculation speed and accuracy that are strongly better than MoM.

As is typical for this kind of problems, once obtained a homogeneous linear system through the discretization operations, the eigensolutions for the electromagnetic field are derived by enforcing the annulment of the determinant of the coefficients' matrix. From a numerical standpoint, the location of these zeros is an ill-conditioned problem. The most reliable results have here been obtained through the evaluation of minima of the squared modulus of the determinant, thus avoiding the very delicate numerical research of complex zeros.

The convergence properties of all the described algorithms have generally appeared quite satisfactory. This allows us to achieve very accurate results, even when the matrix dimensions are maintained rather limited. Consequently, in particular with Nystrom's implementation, the relevant computation time results quite reduced in comparison with other usual numerical techniques (FEM, mode matching, etc.).

Results and discussion

The above described procedure enables the determination of the propagation characteristics for any mode in arbitrarily-shaped dielectric waveguides. The same procedure can easily be applied also for computing the resonant frequencies of resonators of NRD (nonradiative dielectric) type [9]. In fact, the NRD resonators can be viewed as trunks of dielectric waveguides that are short-circuited by two (infinite) parallel metal plates, placed perpendicularly to the symmetry direction at a certain distance apart a (usually chosen less than half the free-space wavelength. In these situations, the β value is fixed by the presence of the plates ($\beta=m\pi/a$, with integer m), and the solution of the integral-equation system is possible only in connection with specific discretized frequencies, that are just at resonance.

Different typical examples have here been considered for these NRD components in order to verify in straightforward manner (also experimentally) the accuracy of the described numerical method. All the BEM data that will now be presented are derived through Nystrom's approach, due to its better performances.

A first class of examples concerns the dielectric structures with rectangular section (transverse dimensions: b and l). In Fig. 2 we show a modal chart for the resonant frequencies f as functions of the length l of parallelepiped NRD resonators, with a fixed choice of the distance between the plates a and of the width b (the case of symmetry with a longitudinal electric ideal wall is presented).

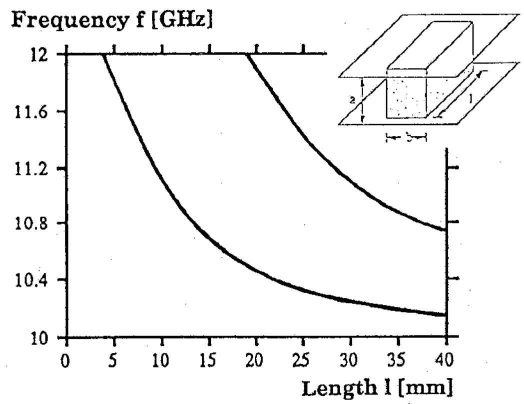
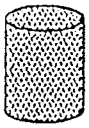


Fig. 2. BEM: behavior of the frequencies f of resonant modes as a function of the length l of an NRD resonator having a rectangular section of dimensions b and l, for a fixed choice of the height a between the plates and of the width b (case of insertion of a longitudinal perfect electric wall). *Parameters:* ε_r*=2.53; b=10 mm; a=12.3 mm.*

As a further significant check of the possibilities of the investigated approach, we have also considered dielectric structures with a circular section, which has been approximated through polygons with a sufficiently high number of sides. A typical example is presented in Table I for a disc NRD resonator (radius R, height a), concerning the frequencies f of the hybrid modes HEM_{npm} in the usual NRD operating range [9] (the indices n,p,m are referred to the angular, radial, and axial variations, respectively). The accuracy of the BEM values is expressed through the relative error, which is evaluated with respect to the results derived by a classical rigorous approach for the circular geometry, based on the straightforward solution of eigenvalue transcendental equations [9]. The agreement between BEM and 'exact' data appears to be remarkable.



Modes HEM npm	BEM Values f [GHz]	Error %
HEM 111	9.225	**– 0.03**
HEM 011	10.698	**0.00**
HEM 021	11.137	**– 0.01**
HEM 211	11.335	**– 0.02**

Table I. BEM: computation of the resonant frequencies *f [GHz]* for the first modes of an NRD resonator having a circular section of radius R and a fixed height a, and relative error with reference to 'exact' data. *Parameters:* ε_r*=2.53; R=11 mm; a=12.3 mm.*

The method has also been tested by considering dielectric structures having non-conventional shapes. In particular, slight perturbations in the geometry of typical cross section of dielectrics have recently found an increasing interest in specific advanced microwave devices; e.g., compact high-performance filters of dual-mode type can be obtained making use of dielectric resonators that have notches or cuts altering their rotation symmetry: the effect of a proper geometrical perturbation on a circular or square resonator consists in the separation of the same degenerate modal frequency into a pair of close frequencies ('quasi-dual modes') [10].

The accurate prediction of the location of such close resonances as a function of the small perturbations, which is a basic requisite for filter design, represents a very difficult task to be solved with numerical procedures. The example that has been presented in Fig. 3 is referred to a notched square-section NRD resonator (side $l=b$, height a, corner's symmetric cut amplitude d): the BEM approach enables us to precisely calculate the variation of the location for the quasi-dual resonant frequencies f_1 and f_2 as a function of the amplitude d of the geometrical perturbation: the greater the notch, the larger the frequency separation. The experimental investigation on such components [10] has shown the agreement with this theoretical behavior.

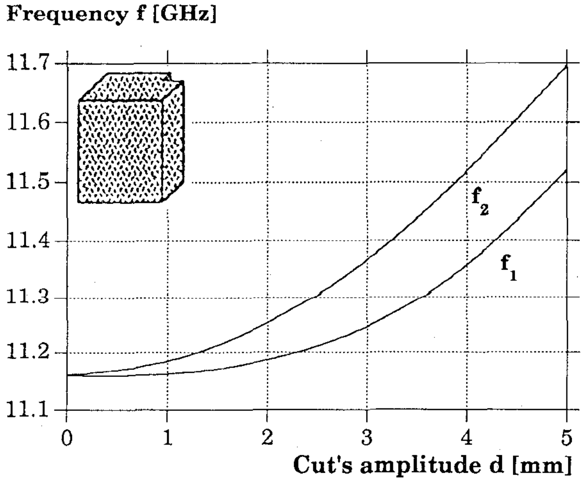


Fig. 3. BEM: variation of the quasi-dual resonant frequencies f_1 and f_2 for a square-section NRD resonator as the amplitude d of the corner's cut varies. *Parameters:* ε_r*=2.53; l=b=10 mm; a=12.3 mm.*

Conclusion

Even though the boundary element method is in principle a well-grounded technique for solving efficiently a variety of electromagnetic problems, in practice a great deal of difficulties can arise from a numerical point of view. The BEM procedure that has here been developed allows us to reach the complete modal characterization of arbitrarily-shaped waveguiding structures making use of a novel formulation, which considerably improves the computational speed, convergence, and accuracy.

These important advantages have been achieved through an analytical development that allows us to profitably avoid derivative operations on the unknowns. To reduce the phenomena of numerical instability, a delicate theoretical evaluation of singular terms has been necessary. The numerical solution has then been possible by developing alternative methods of discretization. An implementation based on Nystrom's method, which avoids numerical integrations, has furnished very fast, economic, and precise results, as confirmed by comparisons with reference theoretical and experimental data for various dielectric structures employed in practice.

Even compared to well-stated rigorous numerical methods (finite elements, mode matching, etc.), this new BEM formulation has proved to be a valid and useful tool for very accurate, efficient, and versatile analysis of a vast class of waveguiding structures.

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